Final Examples

Class outline:

- Trees
- Recursive accumulation
- Regular expressions
- Interpreters

Trees

Tree abstractions

In Python, using a class:

```
class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
        self.branches = list(branches)

def is_leaf(self):
    return not self.branches
```

In Scheme, using procedures to build a data abstraction:

```
(define (tree label branches)
    (cons label branches))

(define (label t) (car t))

(define (branches t) (cdr t))

(define (is-leaf t) (null? (branches t)))
```

Tree-structured data

A tree is a recursive structure, where each branch may itself be a tree.

```
[5, [6, 7], 8, [[9], 10]]
(+ 5 (- 6 7) 8 (* (- 9) 10))
(S
   (NP (JJ Short) (NNS cuts))
   (VP (VBP make)
       (NP (JJ long) (NNS delays)))
   (...))
<u1>
   Midterm <strong>1</strong>
   Midterm <strong>2</strong>
```

Tree processing often involves recursive calls on subtrees.

Implement bigs, which takes a Tree instance t containing integer labels. It returns the number of nodes in t whose labels are larger than all labels of their ancestor nodes.

```
def bigs(t):
    """Return the number of nodes in t that are larger than all their ancestors.

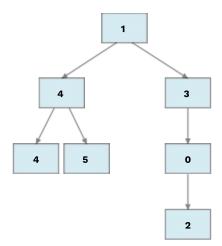
>>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])))
>>> bigs(a)
4
    """
```

1. Understand the question and function signature.

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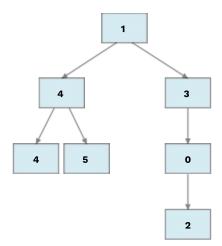
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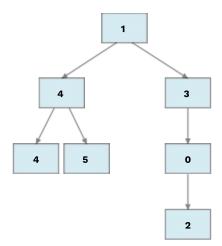
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- 3. Work through the examples and make observations.



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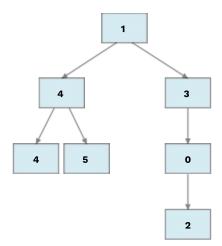
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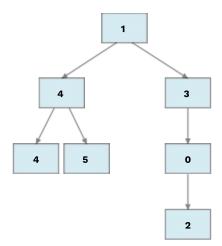
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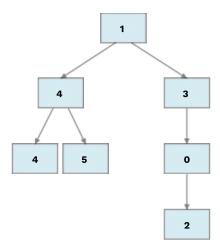
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4. Consider what you expect to see in the solution.

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4
"""
```

4. Consider what you expect to see in the solution.

Typical tree processing structure?

```
if t.is_leaf():
    return ____
else:
    return ____([___ for b in t.branches])
```

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def bigs(t):
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    """
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Typical tree processing structure?

```
if t.is_leaf():
    return ____
else:
    return ____([___ for b in t.branches])
```

X That won't work, since we need to know about ancestors.

```
def bigs(t):
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4. Consider what you expect to see in the solution.

Some code that increments the total count

```
1 + _____
```

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    """
```

4. Consider what you expect to see in the solution.

Some code that increments the total count

```
1 + _____
```

Some way of tracking ancestor labels or max of ancestors seen so far.

```
if node.label > max(ancestors):

if node.label > max_ancestor:
```

```
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"""
```

5. Check out the provided template.

```
def f(a, x):
    if _____:
        return 1 + _____:
        return ____:
        return
```

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>>> a = Tree(1, [Tree(4, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(2)])])))
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- 5. Check out the provided template.
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```
def f(a, x):
    if ______:
        return 1 + _____ # Increment total
    else:
        return _____
return _____
```

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def bigs(t):
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- 5. Check out the provided template.
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```
def f(a, x):
    if ______: # Track the largest ancestor
        return 1 + _____ # Increment total
    else:
        return _____
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```
# a is the current subtree, x is the largest ancestor

def f(a, x):
    if ______: # Track the largest ancestor
        return 1 + ______ # Increment total
    else:
        return _____
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    """
```

8. Finish filling in the skeleton.

```
def f(a, x):
    if a.label > x:
        return 1 + sum([f(b, a.label) for b in a.branches])
    else:
        return sum([f(b, x) for b in a.branches])
return f(t, t.label - 1)
```

```
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```

Recursive accumulation

Initialize some data structure to an empty/zero value, and populate it as you go.

Solving smalls

Implement smalls, which takes a Tree instance t containing integer labels. It returns the non-leaf nodes in t whose labels are smaller than any labels of their descendant nodes.

```
def smalls(t):
    """Return the non-leaf nodes in t that are smaller than all their descendants.

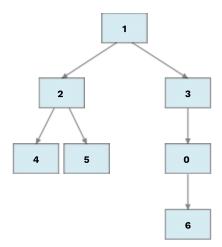
>>> a = Tree(1, [Tree(2, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(6)])])))
>>> sorted([t.label for t in smalls(a)])
    [0, 2]
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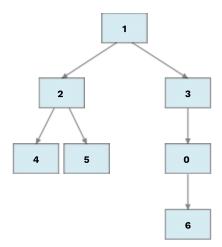
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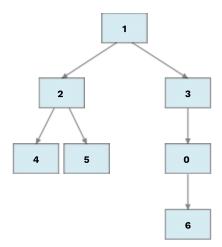
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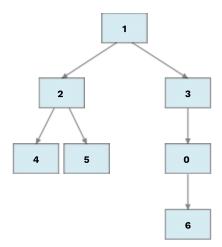
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4. Consider what you expect to see in the solution.

Something which finds the smallest value in a subtree

```
min(____)
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t.label < smallest
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```

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Something which finds the smallest value in a subtree

```
min(____)
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Something which compares smallest to current

```
t.label < smallest
```

Something which adds a subtree to a list

```
___.append(t)
```

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- 5. Check out the provided template.
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```
result = []
def process(t):
    if t.is_leaf():
        return
    else:
        smallest = ______ # Finds smallest
        if ______:
        return min(smallest, t.label)
process(t)
return result
```

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    """
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- 5. Check out the provided template.
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```
result = []
def process(t):
    if t.is_leaf():
        return
    else:
        smallest = _____ # Finds smallest
        if _____: # Compares smallest

        return min(smallest, t.label)
process(t)
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```

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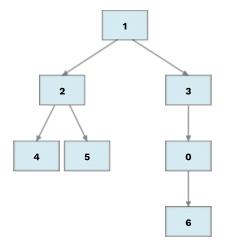
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```

8. Finish filling in the skeleton.

```
result = []
def process(t):
    if t.is_leaf():
        return t.label
    else:
        smallest = min([process(b) for b in t.branches])
        if t.label < smallest:
            result.append(t)
        return min(smallest, t.label)
process(t)
return result</pre>
```

```
def smalls(t):
    """Return the non-leaf nodes in t that are smaller than all their descendants.
    >>> a = Tree(1, [Tree(2, [Tree(4), Tree(5)]), Tree(3, [Tree(0, [Tree(6)])]))
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            if t.label < smallest:
                result.append(t)
            return min(smallest, t.label)
    process(t)
    return result</pre>
```

8. Check your work!



Regular expressions

Matching patterns

Which strings are matched by each regular expression?

Expressions:	abc	cab	bac	baba	ababca	aabcc	abba
[abc]*							
a*b*c*							
ab [bc]*							
(a[bc]+)+a?							
(ab ba)+ (ab [bc])?							

Matching patterns

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Expressions:	abc	cab	bac	baba	ababca	aabcc	abba
[abc]*	V	V	V	\mathscr{O}	V	V	\mathcal{O}
a*b*c*	\emptyset	×	×	×	×	\mathcal{O}	×
ab [bc]*	×	×	×	×	×	×	×
(a[bc]+)+a?	V	×	×	×	V	×	\mathcal{O}
(ab ba)+ (ab [bc])?	\mathscr{O}	×	\mathcal{O}	Ø.	×	×	Ø

Interpreters

```
(define x (+ 1 2))

(define (f y) (+ x y))

(f (if (> 3 2) 4 5))
```

```
(define x (+ 1 2))
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```
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((f (if (> 3 2) 4 5)))
```

```
(define x (+ 1 2))
(define (f y) (+ x y))
((f (if (> 3 2) 4 5)))
```

```
(define x (+ 1 2))
(define (f y) (+ x y))
((f (if ((> 3 2) 4 5)))
```

```
(define x (+ 1 2))
(define (f y) (+ x y))
((f (if (> 3 2) 4 5)))
```

```
(define x (+ 1 2))
(define (f y) (+ x y))
(f (if (> 3 2) 4 5))
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```
(define x (+ 1 2))
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