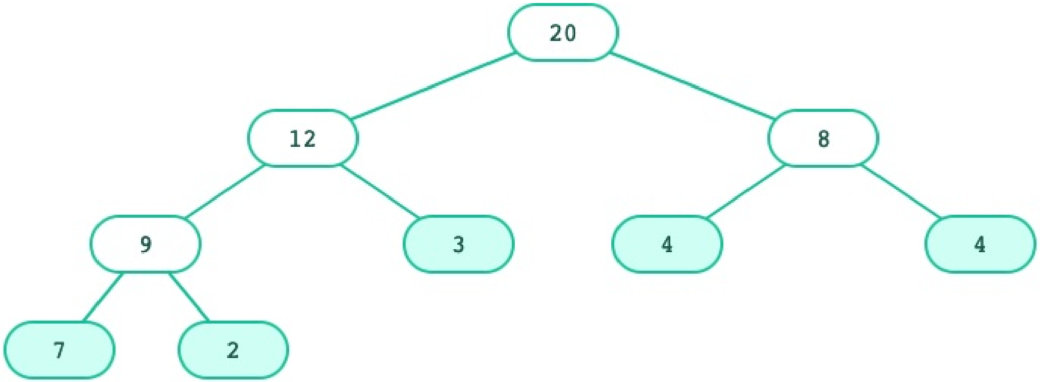
Trees

Class outline:

- Trees
- Tree class
- Tree processing
- Tree creation
- Tree mutation

Trees

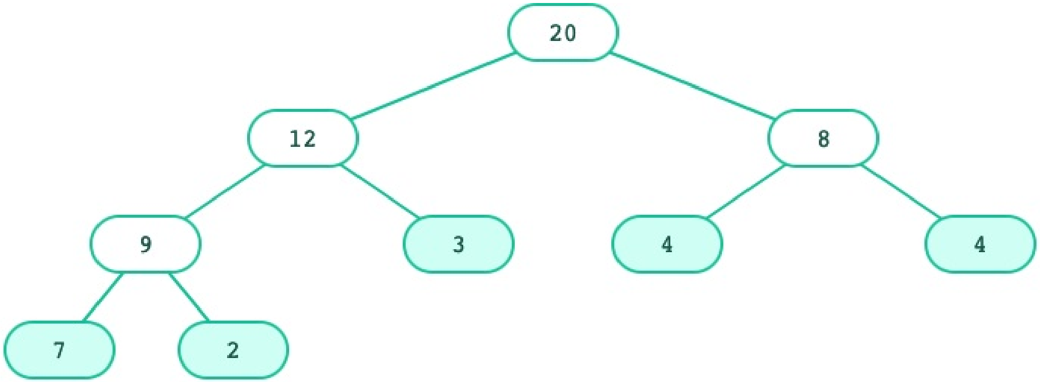
Trees



Recursive description

- A tree has a **root label** and a list of **branches**
- Each **branch** is itself a tree
- A tree with zero branches is called a **leaf**
- A tree starts at the **root**

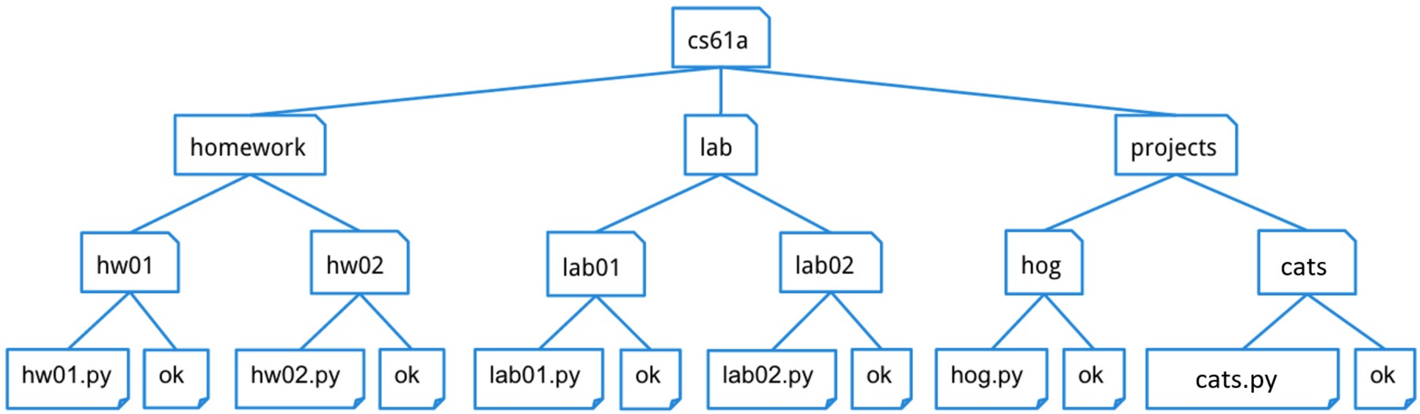
Trees



Recursive description	**Relative description**
• A tree has a **root label** and a list of **branches** • Each **branch** is itself a tree • A tree with zero branches is called a **leaf** • A tree starts at the **root**	• Each location in a tree is called a **node** • Each node has a **label** that can be any value • One node can be the **parent/child** of another • The top node is the **root node**

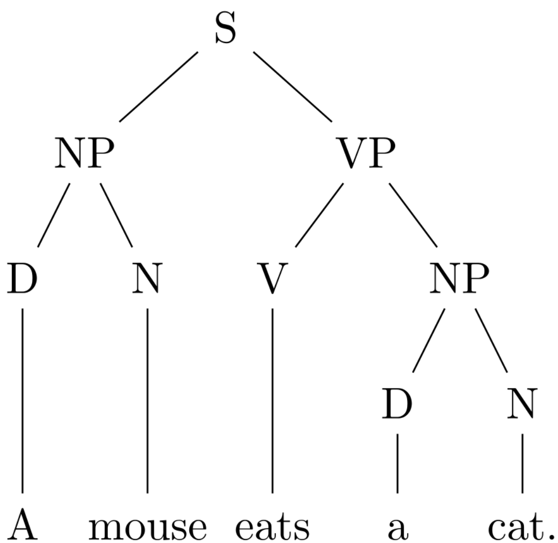
Trees, trees, everywhere!

Directory structures



Parse trees

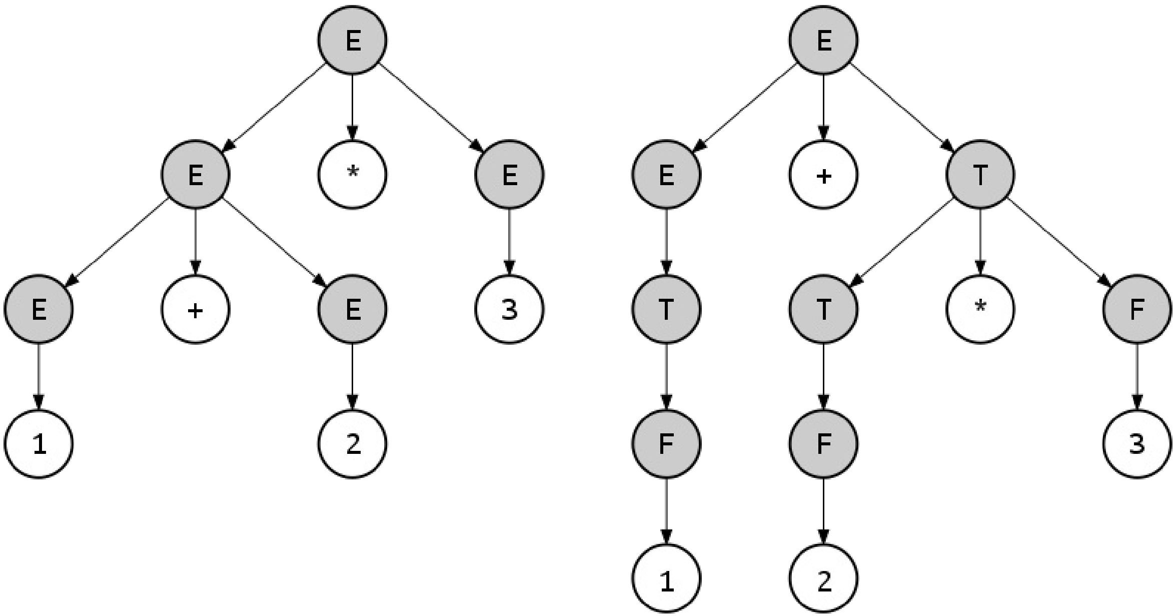
For natural languages...



Key: S = Sentence, NP = Noun phrase, D = Determiner, N = Noun, V = Verb, VP = Verb Phrase

Parse trees

For programming languages, too...



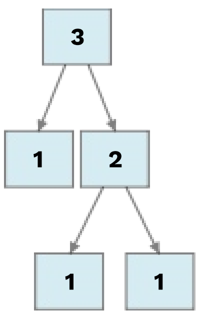
Key: E = expression

Tree class

A Tree object

A Tree is an object composed of other Tree objects, so its constructor must have a way of passing in children Trees.

Our approach:




```
t = Tree(3, [
            Tree(1),
            Tree(2, [
                    Tree(1),
                    Tree(1)
                    ])
            ])
```

The Tree object (cont'd)

A Tree should store these instance variables:

`label`	The root label of the tree
`branches`	A list of branches (subtrees) of the tree

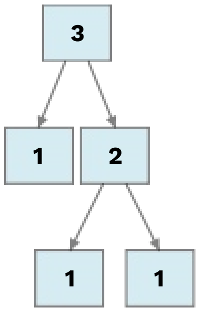
And expose this instance method:

`is_leaf`	Returns a boolean indicating if tree is a leaf

```
t = Tree(3, [
            Tree(1),
            Tree(2, [
                    Tree(1),
                    Tree(1)
                    ])
            ])

t.label
t.is_leaf()
```

```
t.branches[0].is_leaf()
```



The Tree object (cont'd)

A Tree should store these instance variables:

`label`	The root label of the tree
`branches`	A list of branches (subtrees) of the tree

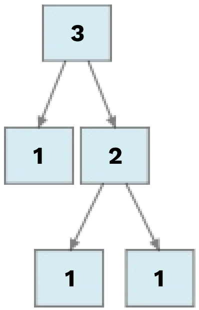
And expose this instance method:

`is_leaf`	Returns a boolean indicating if tree is a leaf

```
t = Tree(3, [
            Tree(1),
            Tree(2, [
                    Tree(1),
                    Tree(1)
                    ])
            ])

t.label                         # 3
t.is_leaf()
```

```
t.branches[0].is_leaf()
```



The Tree object (cont'd)

A Tree should store these instance variables:

`label`	The root label of the tree
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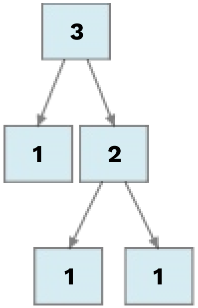
```
t = Tree(3, [
            Tree(1),
            Tree(2, [
                    Tree(1),
                    Tree(1)
                    ])
            ])

t.label                         # 3
t.is_leaf()                     # False
```

```
t.branches[0].is_leaf()
```



The Tree object (cont'd)

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`label`	The root label of the tree
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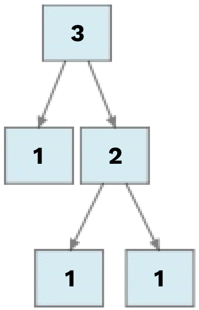
And expose this instance method:

`is_leaf`	Returns a boolean indicating if tree is a leaf

```
t = Tree(3, [
            Tree(1),
            Tree(2, [
                    Tree(1),
                    Tree(1)
                    ])
            ])

t.label                  # 3
t.is_leaf()              # False
```

```
t.branches[0].is_leaf()   # True
```



The Tree class

```
t = Tree(3, [Tree(1), Tree(2, [Tree(1), Tree(1)])])

t.label                    # 3
t.is_leaf()                # False
t.branches[0].is_leaf()    # True
```

How could we write the class definition for `Tree`?

The Tree class

```
t = Tree(3, [Tree(1), Tree(2, [Tree(1), Tree(1)])])

t.label                 # 3
t.is_leaf()             # False
t.branches[0].is_leaf() # True
```

How could we write the class definition for `Tree`?

```
class Tree:
    def __init__(self, label, branches=[]):
        self.label = label
        self.branches = list(branches)

    def is_leaf(self):
        return not self.branches
```

A fancier Tree

This is what assignments actually use:

```
class Tree:

    def __init__(self, label, branches=[]):
        self.label = label
        for branch in branches:
            assert isinstance(branch, Tree)
        self.branches = list(branches)

    def is_leaf(self):
        return not self.branches

    def __repr__(self):
        if self.branches:
            branch_str = ', ' + repr(self.branches)
        else:
            branch_str = ''
        return 'Tree({0}{1})'.format(self.label, branch_str)

    def __str__(self):
        return '\n'.join(self.indented())

    def indented(self):
        lines = []
        for b in self.branches:
            for line in b.indented():
                lines.append('  ' + line)
        return [str(self.label)] + lines
```

It's built in to code.cs61a.org, and you can `draw()` any `Tree`!

Tree processing

Tree processing

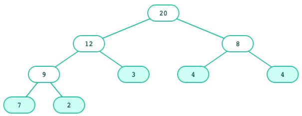
A tree is a recursive structure.

Each tree has:

- A label
- 0 or more branches, each a tree

Recursive structure implies recursive algorithm!

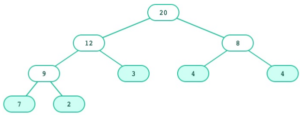
Counting leaves




```python
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if

    else:
```

What's the base case? What's the recursive call?

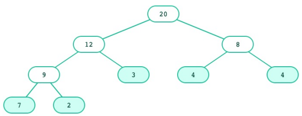
Counting leaves




```
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if t.is_leaf():

    else:
```

What's the base case? What's the recursive call?

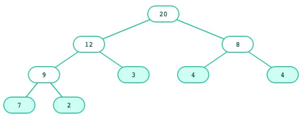
Counting leaves




```
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if t.is_leaf():
        return 1
    else:
```

What's the base case? What's the recursive call?

Counting leaves




```
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if t.is_leaf():
        return 1
    else:
        leaves_under = 0
        for b in t.branches:
            leaves_under += count_leaves(b)
        return leaves_under
```

What's the base case? What's the recursive call?

Counting leaves (cont'd)

The `sum()` function sums up the items of an iterable.

```
sum([1, 1, 1, 1])    # 4
```

Counting leaves (cont'd)

The `sum()` function sums up the items of an iterable.

```
sum([1, 1, 1, 1])    # 4
```

That leads to this shorter function:

```
def count_leaves(t):
    """Returns the number of leaf nodes in T."""
    if t.is_leaf():
        return 1
    else:
        branch_counts = [count_leaves(b) for b in t.branches]
        return sum(branch_counts)
```

Exercise: Printing trees

```
def print_tree(t, indent=0):
    """Prints the labels of T with depth-based indent.
    >>> t = Tree(3, [Tree(1), Tree(2, [Tree(1), Tree(1)])])
    >>> print(t)
    3
      1
      2
        1
        1
    """
```

Exercise: Printing trees (solution)

```
def print_tree(t, indent=0):
    """Prints the labels of T with depth-based indent.
    >>> t = Tree(3, [Tree(1), Tree(2, [Tree(1), Tree(1)])])
    >>> print(t)
    3
      1
      2
        1
        1
    """
    print(indent * " " + t.label)
    for b in t.branches:
        print_tree(b, indent + 2)
```

Exercise: List of leaves

```
def leaves(t):
    """Return a list containing the leaf labels of T.
    >>> t = Tree(20, [Tree(12, [Tree(9, [Tree(7), Tree(2)]), Tree(3
    >>> leaves(t)
    [7, 2, 3, 4, 4]
    """
```

Hint: If you sum a list of lists, you get a list containing the elements of those lists. The sum function takes a second argument, the starting value of the sum.

```
sum([ [1], [2, 3], [4] ], []) # [1, 2, 3, 4]
sum([ [1] ], []) # [1]
sum([ [[1]], [2] ], []) # [[1], 2]
```

Exercise: List of leaves (Solution)

```
def leaves(t):
    """Return a list containing the leaf labels of T.
    >>> t = Tree(20, [Tree(12, [Tree(9, [Tree(7), Tree(2)]), Tree(1
    >>> leaves(t)
    [7, 2, 3, 4, 4]
    """
    if t.is_leaf():
        return [t.label]
    else:
        leaf_labels = [leaves(b) for b in t.branches]
        return sum(leaf_labels, [])
```

Exercise: Counting paths

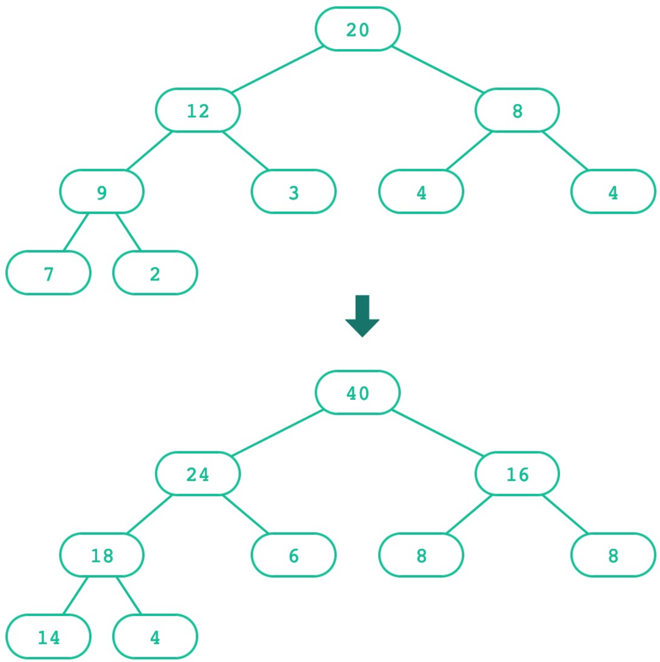
```
def count_paths(t, total):
    """Return the number of paths from the root to any node in T
    for which the labels along the path sum to TOTAL.

    >>> t = Tree(3, [Tree(-1), Tree(1, [Tree(2, [Tree(1)]), Tree(3)]), Tree(1, [
    >>> count_paths(t, 3)
    2
    >>> count_paths(t, 4)
    2
    >>> count_paths(t, 5)
    0
    >>> count_paths(t, 6)
    1
    >>> count_paths(t, 7)
    2
    """
```

Exercise: Counting paths (solution)

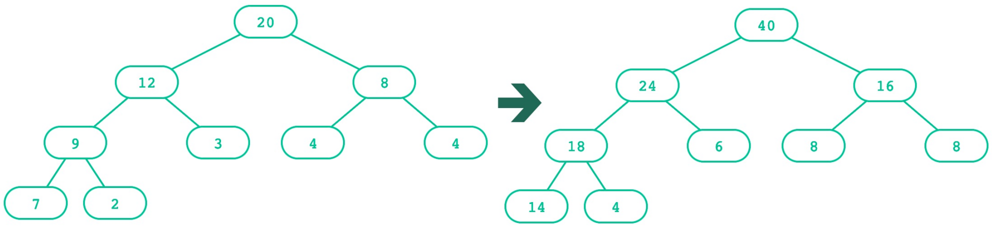
```
def count_paths(t, total):
    """Return the number of paths from the root to any node in T
    for which the labels along the path sum to TOTAL.

    >>> t = Tree(3, [Tree(-1), Tree(1, [Tree(2, [Tree(1)]), Tree(3)]), Tree(1, [
    >>> count_paths(t, 3)
    2
    >>> count_paths(t, 4)
    2
    >>> count_paths(t, 5)
    0
    >>> count_paths(t, 6)
    1
    >>> count_paths(t, 7)
    2
    """
    if t.label == total:
        found = 1
    else:
        found = 0
    return found + sum([count_paths(b, total - t.label) for b in t.branches])
```

Creating trees

A function that creates a tree from another tree is also often recursive.



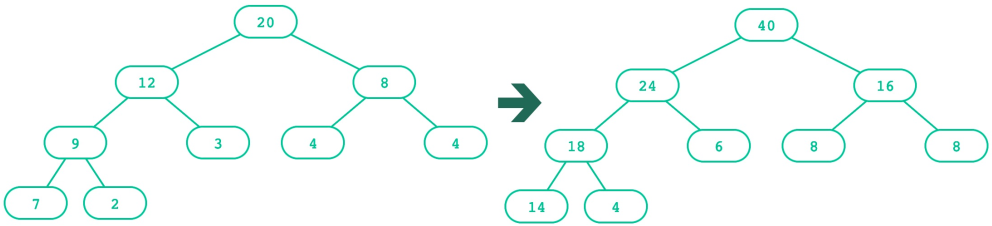
Creating trees: Doubling labels




```
def double(t):
    """Returns a tree identical to T, but with all la
    if

    else:
```

What's the base case? What's the recursive call?

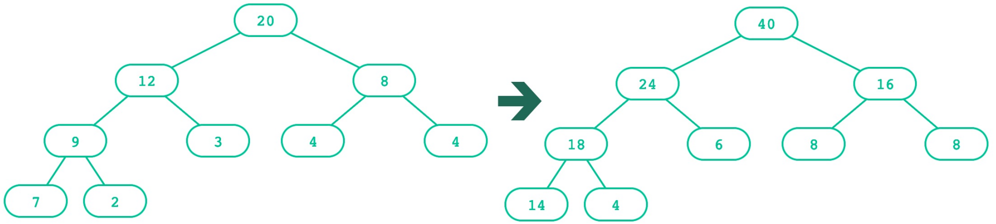
Creating trees: Doubling labels




```
def double(t):
    """Returns a tree identical to T, but with all la
    if t.is_leaf():

    else:
```

What's the base case? What's the recursive call?

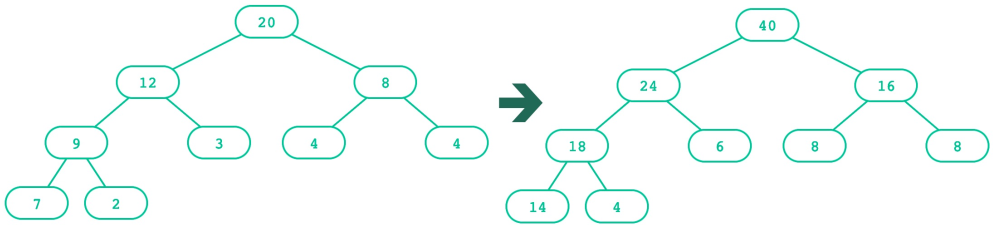
Creating trees: Doubling labels




```
def double(t):
    """Returns a tree identical to T, but with all la
    if t.is_leaf():
        return Tree(t.label * 2)
    else:
```

What's the base case? What's the recursive call?

Creating trees: Doubling labels




```
def double(t):
    """Returns a tree identical to T, but with all la
    if t.is_leaf():
        return Tree(t.label * 2)
    else:
        return Tree(t.label * 2,
            [double(b) for b in t.branches])
```

What's the base case? What's the recursive call?

Creating trees: Doubling labels

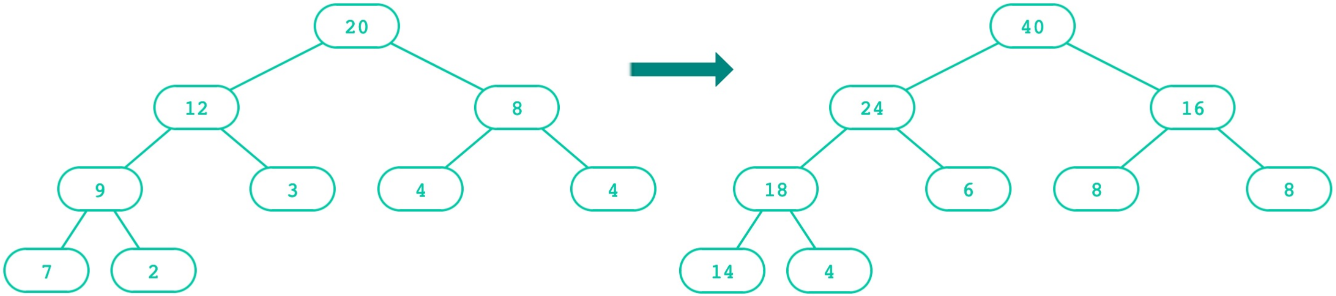
A shorter solution:

```
def double(t):
    """Returns the number of leaf nodes in T."""
    return Tree(t.label * 2,
                [double(b) for b in t.branches])
```

Explicit base cases aren't always necessary in the final code, but it's useful to think in terms of base case vs. recursive case when learning.

Tree mutation

Doubling a Tree

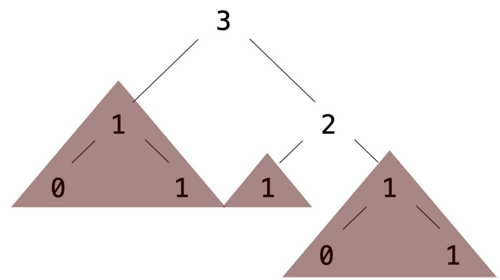



```
def double(t):
    """Doubles every label in T, mutating T.
    >>> t = Tree(1, [Tree(3, [Tree(5)]), Tree(7)])
    >>> double(t)
    >>> t
    Tree(2, [Tree(6, [Tree(10)]), Tree(14)])
    """
    t.label = t.label * 2
    for b in t.branches:
        double(b)
```

Exercise: Pruning trees

Removing subtrees from a tree is called **pruning**.

Always prune branches before recursive processing.

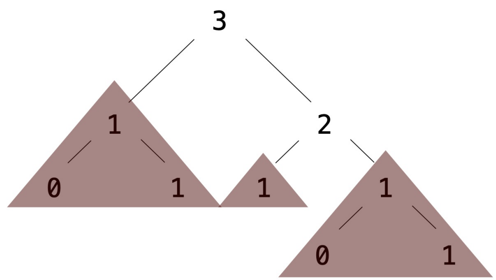



```
def prune(t, n):
    """Prune all sub-trees whose label is n.
    >>> t = Tree(3, [Tree(1, [Tree(0), Tree(1)]), Tree(2, [Tree(1),
    >>> prune(t, 1)
    >>> t
    Tree(3, [Tree(2)])
    """
    t.branches = [___ for b in t.branches if ___]
    for b in t.branches:
        prune(___, ___)
```

Exercise: Pruning trees (Solution)

Removing subtrees from a tree is called **pruning**.

Always prune branches before recursive processing.




```
def prune(t, n):
    """Prune all sub-trees whose label is n.
    >>> t = Tree(3, [Tree(1, [Tree(0), Tree(1)]), Tree(2, [Tree(1),
    >>> prune(t, 1)
    >>> t
    Tree(3, [Tree(2)])
    """
    t.branches = [b for b in t.branches if b.label !=n]
    for b in t.branches:
        prune(b, n)
```