

Environments

Class outline:

- Multiple environments
- Environments for HOFs
- Local names
- Function composition
- Currying

Multiple Environments

Life cycle of a function

Def statement

```
def square ( x ) :  
    return x * x
```

What happens?

- A new function is created!
- Name bound to that function in the current frame.

Call expression

```
square ( 2 + 2 )
```

- Operator & operands evaluated
- Function (value of operator) called on arguments (values of operands)

Calling/applying

```
def square( x )
```

```
▶ 16
```

```
4 ▶
```

- A new frame is created!
- Parameters bound to arguments
- Body is executed in that new environment

A nested call expression

- 1.
- 2.
- 3.

```
def square(x):  
    return x * x  
square(square(3))
```


A nested call expression

- 1.
- 2.
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next

```
def square(x):  
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A nested call expression

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prev

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```
def square(x):  
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Global frame

square | • ----> func square(x) [parent=Global]

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A nested call expression

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```
square( square(3) )
```

```
func square(x)
```

A nested call expression

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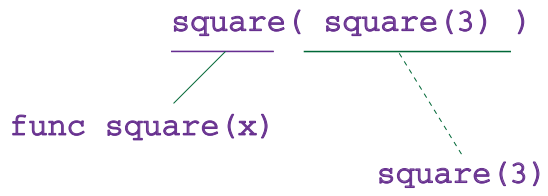
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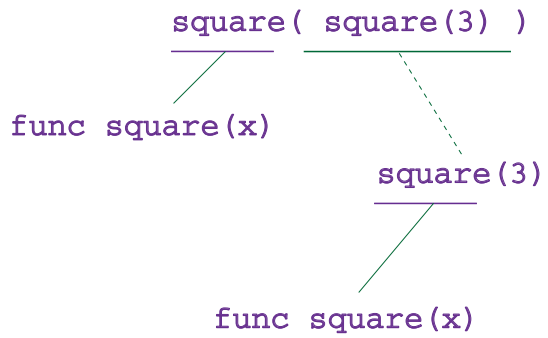
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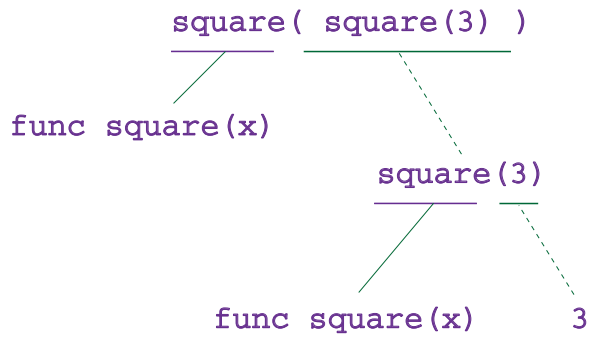
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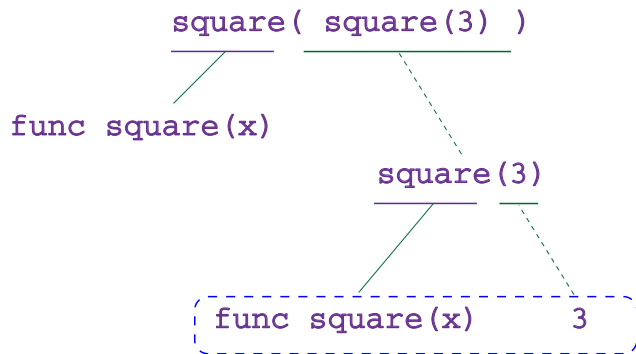
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A nested call expression

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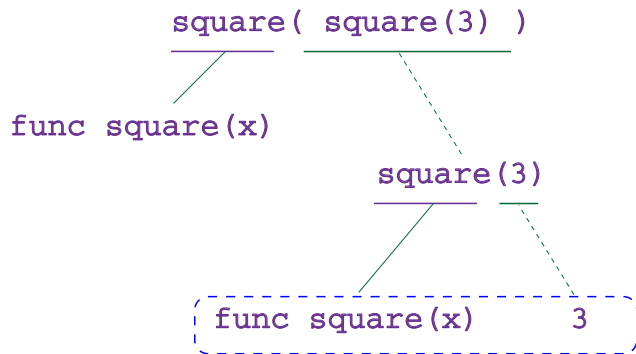
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```

Global frame

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f1: square [parent=Global]

x | 3



A nested call expression

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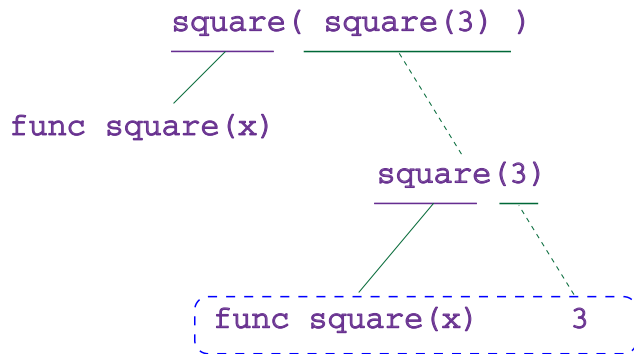
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Global frame

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f1: square [parent=Global]

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A nested call expression

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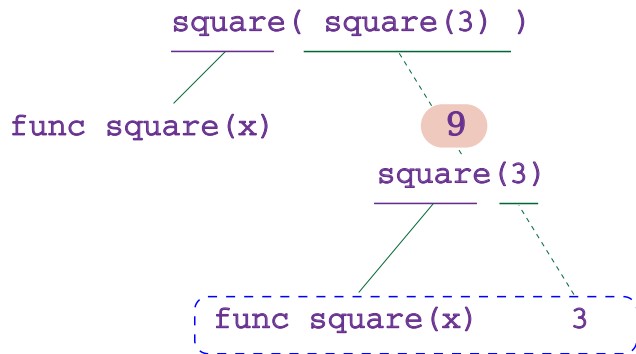
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square(square(3))
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Global frame

square | • ----> func square(x) [parent=Global]

f1: square [parent=Global]

Return value | $\begin{array}{r} \times 3 \\ \hline 9 \end{array}$



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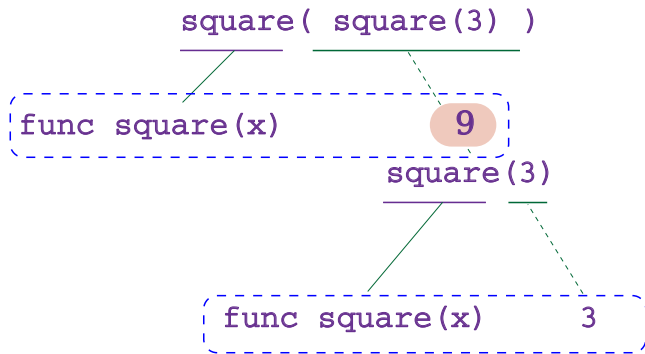
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Global frame

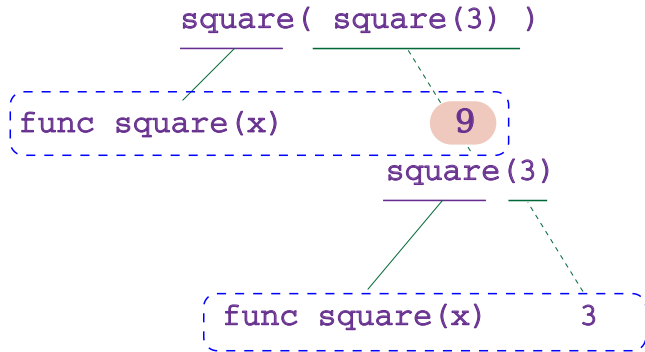
square | • ----> func square(x) [parent=Global]

f1: square [parent=Global]

Return value $\begin{array}{r} \times 3 \\ \hline 9 \end{array}$

f2: square [parent=Global]

|



A nested call expression

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Global frame

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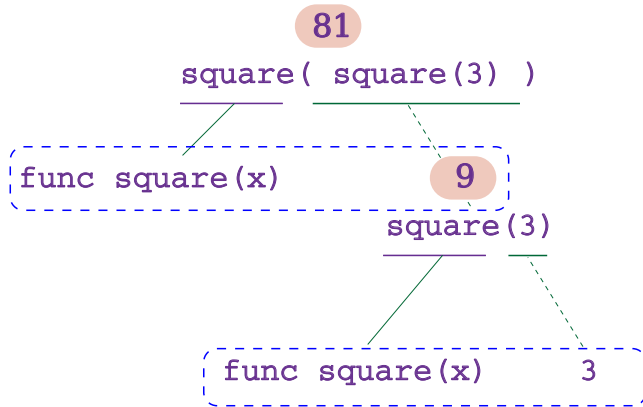
f1: square [parent=Global]

Return value $\begin{array}{r} \times 3 \\ \hline 9 \end{array}$

f2: square [parent=Global]

|

Return value $\begin{array}{|c} \times 9 \\ \hline 81 \end{array}$



Multiple environments in one diagram!

```
def square(x):  
    return x * x  
  
square(square(3))
```

Global frame

square $\boxed{\bullet}$ ----> func square(x) [parent=Global]

f1: square [parent=Global]

Return value $\begin{array}{r} \times 3 \\ \hline 9 \end{array}$

f2: square [parent=Global]

Return value $\begin{array}{r} \times 9 \\ \hline 81 \end{array}$

An environment is a sequence of frames.

Multiple environments in one diagram!

```
def square(x):  
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square(square(3))
```

1 Global frame

square $\boxed{\bullet}$ -----> func square(x) [parent=Global]

f1: square [parent=Global]

Return value $\begin{array}{r} \times 3 \\ \hline 9 \end{array}$

f2: square [parent=Global]

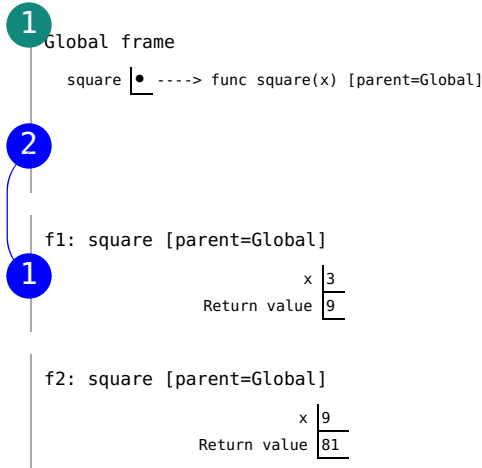
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An environment is a sequence of frames.

- Environment: Global frame

Multiple environments in one diagram!

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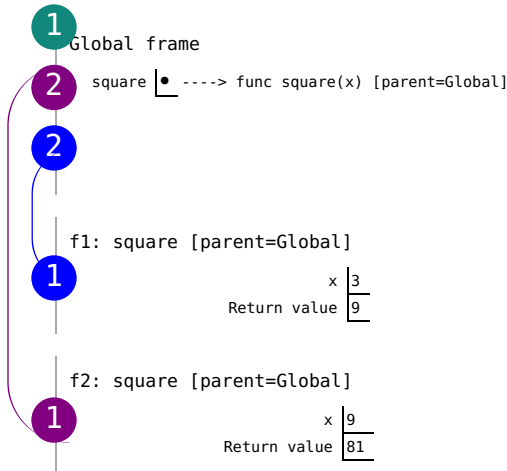


An environment is a sequence of frames.

- Environment: Global frame
- Environment: Local frame (f1), then global frame

Multiple environments in one diagram!

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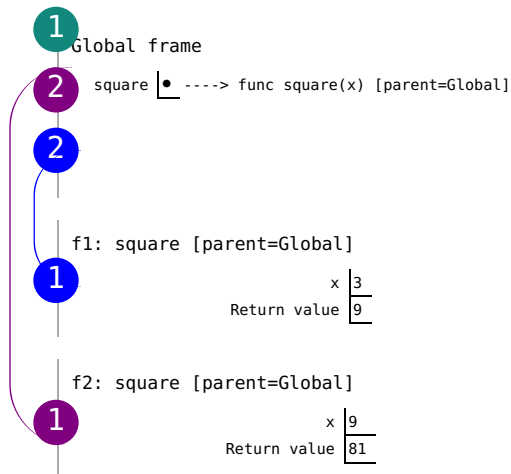


An environment is a sequence of frames.

- Environment: Global frame
- Environment: Local frame (f1), then global frame
- Environment: Local frame (f2), then global frame

Names have no meanings without environments

```
def square(x):  
    return x * x  
  
square(square(3))
```



Every expression is evaluated in the context of an environment.

A name evaluates to the value bound to that name in the earliest frame of the current environment in which that name is found.

Names have different meanings in different environments

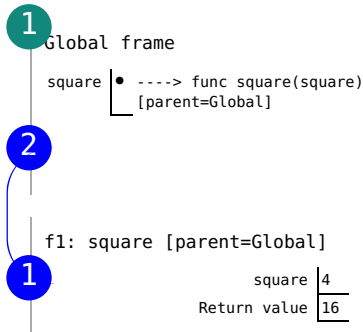
```
def square(square) :  
    return square * square  
  
square(4)
```

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Names have different meanings in different environments

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def square(square) :  
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Environments for higher-order functions

Review: Higher-order functions

A higher-order function is either...

- A function that takes a function as an argument value
`summation(5, lambda x: x**2)`
- A function that returns a function as a return value
`make_adder(3)(1)`

Functions are first class: Functions are values in Python.

Example: Apply twice

```
def apply_twice(f, x):  
    return f(f(x))  
  
def square(x):  
    return x ** 2  
  
apply_twice(square, 3)
```



[View in PythonTutor](#)

Arguments bound to functions



Arguments bound to functions



Arguments bound to functions

?

?



Environments for nested definitions

Example: Make texter

```
def make_texter(emoji):  
    def texter(text):  
        return emoji + text + emoji  
    return texter
```

```
happy_text = make_texter("😊")  
result = happy_text("lets go to the beach!")
```



[View in PythonTutor](#)

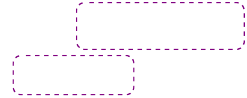
Environments for nested def statements



Environments for nested def statements



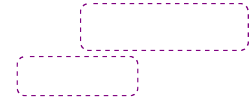
- Every user-defined **function** has a parent frame
- The parent of a **function** is the **frame in which it was defined**



Environments for nested def statements



- Every user-defined **function** has a parent frame
- The parent of a **function** is the **frame in which it was defined**
- Every local **frame** has a parent frame
- The parent of a **frame** is the **parent of the called function**



Environments for nested def statements



- Every user-defined **function** has a parent frame
- The parent of a **function** is the **frame in which it was defined**
- Every local **frame** has a parent frame
- The parent of a **frame** is the **parent of the called function**
- An environment is a **sequence of frames**.

1

How to draw an environment diagram

When a function is defined:

1. Create a function value:

```
func <name>(<formal parameters>) [parent=<label>]
```

2. Its parent is the current frame.
3. Bind `<name>` to the function value in the current frame

When a function is called:

1. Add a local frame, titled with the `<name>` of the function being called.
2. Copy the parent of the function to the local frame:
`[parent=>label<]`
3. Bind the `<formal parameters>` to the arguments in the local frame.
4. Execute the body of the function in the environment that starts with the local frame.

Local names

Example: Thingy Bobber

```
def thingy(x, y):  
    return bobber(y)  
  
def bobber(a):  
    return a + y  
  
result = thingy("ma", "jig")
```

What do you think will happen?

Example: Thingy Bobber

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def thingy(x, y):  
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def bobber(a):  
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result = thingy("ma", "jig")
```

What do you think will happen?



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Local name visibility

Local names are not visible to other (non-nested) functions.



- An environment is a sequence of frames.
- The environment created by calling a top-level function consists of one local frame followed by the global frame.

1

Function Composition

Example: Composer

```
def happy(text):  
    return "😊" + text + "😊"  
  
def sad(text):  
    return "😞" + text + "😞"  
  
def composer(f, g):  
    def composed(x):  
        return f(g(x))  
    return composed  
  
msg1 = composer(sad, happy)("cs61a!")  
msg2 = composer(happy, sad)("eecs16a!")
```

What do you think will happen?

Example: Composer (Part 2)

One of the composed functions could itself be an HOF...

```
def happy(text):
    return "😊" + text + "😊"

def sad(text):
    return "😞" + text + "😞"

def make_texter(emoji):
    def texter(text):
        return emoji + text + emoji
    return texter

def composer(f, g):
    def composed(x):
        return f(g(x))
    return composed

composer(happy, make_texter("😞"))('snow day!')
```




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Composer 2 expression tree

```
composer(happy, make_texter("🌨"))("snow day!")
```

Composer 2 expression tree

```
composer(happy, make_texter("🌨️"))("snow day!")
```



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Composer 2 expression tree

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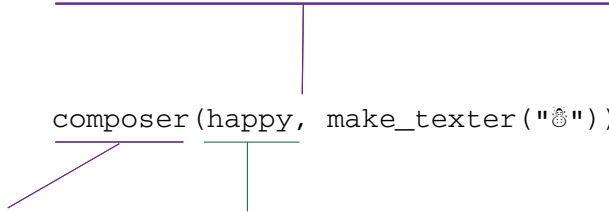
```
func composer(f, g)
```

Composer 2 expression tree

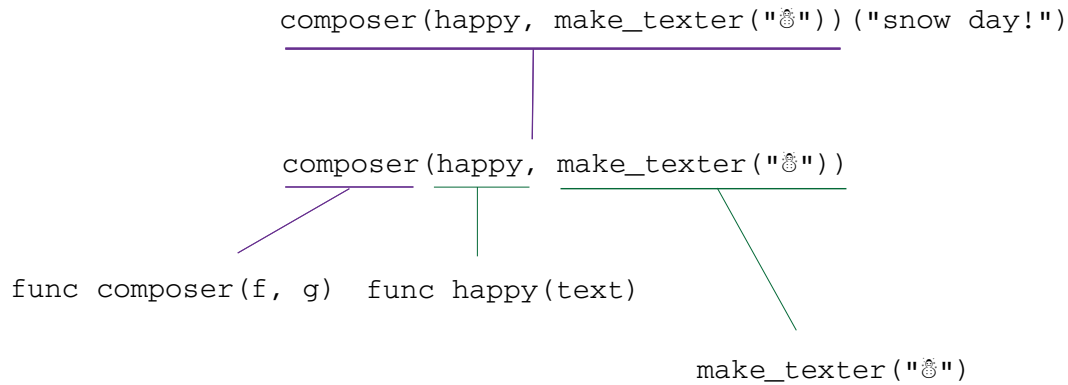
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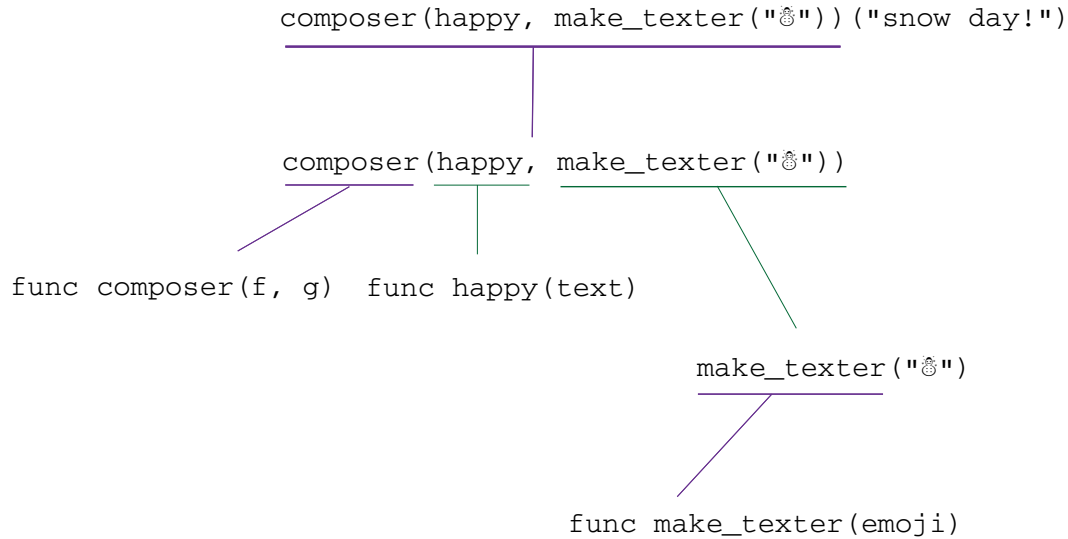
`func composer(f, g) func happy(text)`



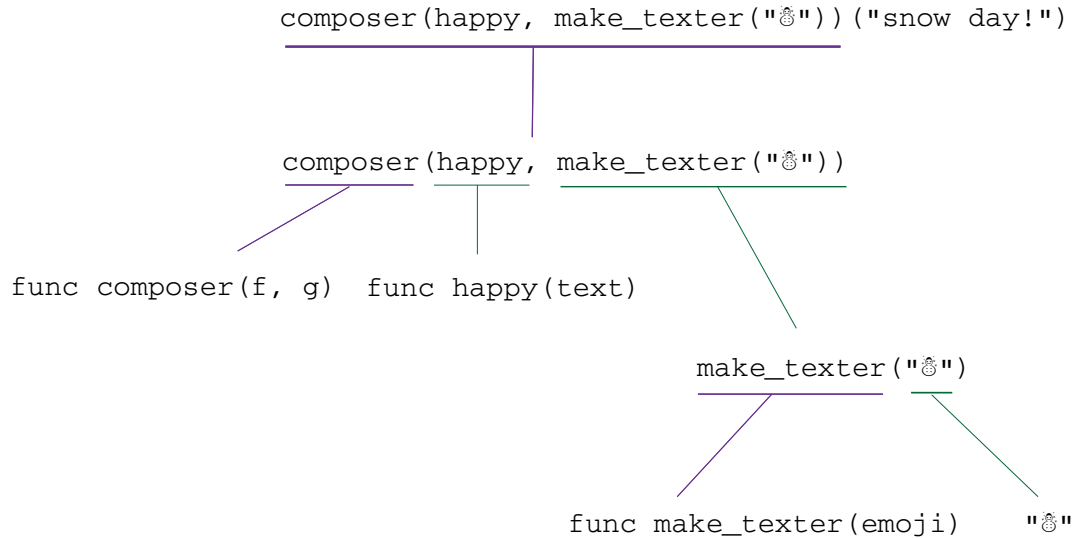
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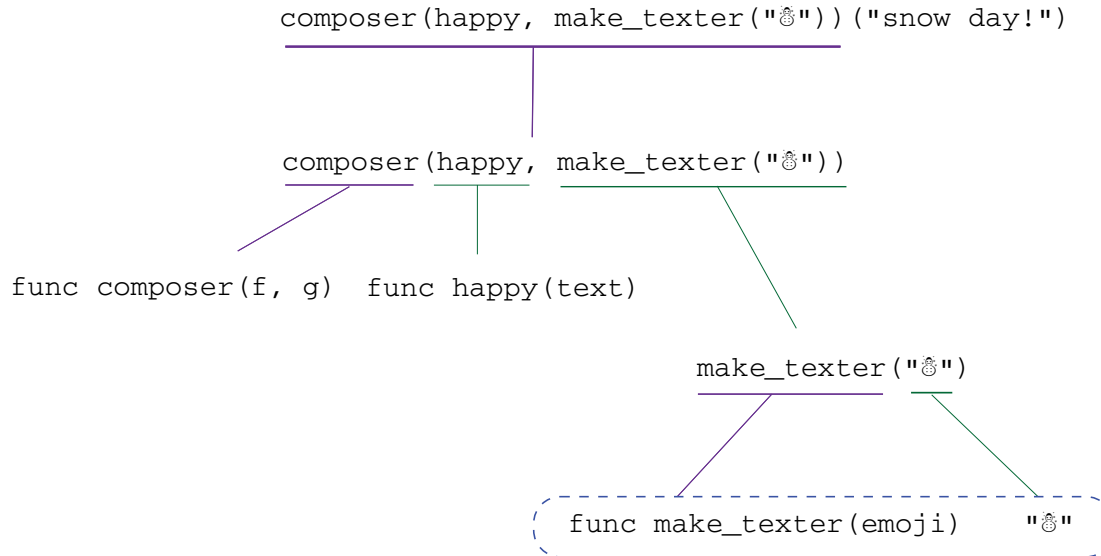
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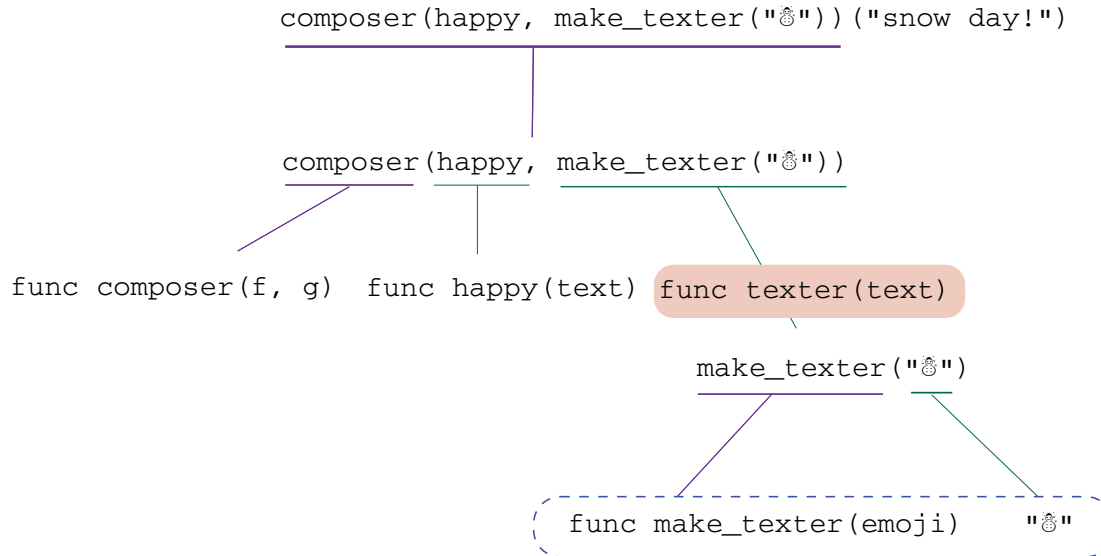
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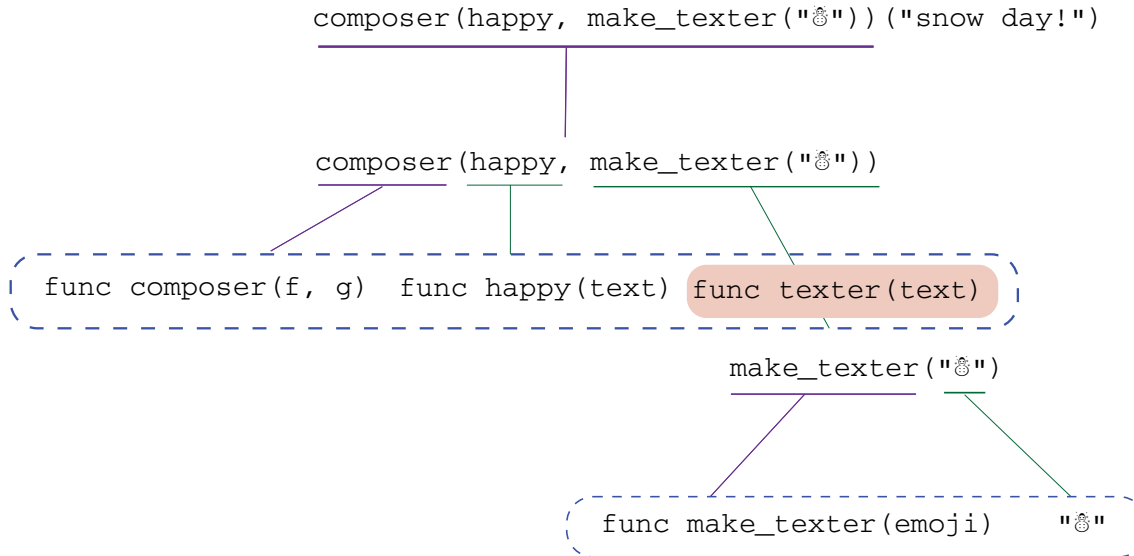
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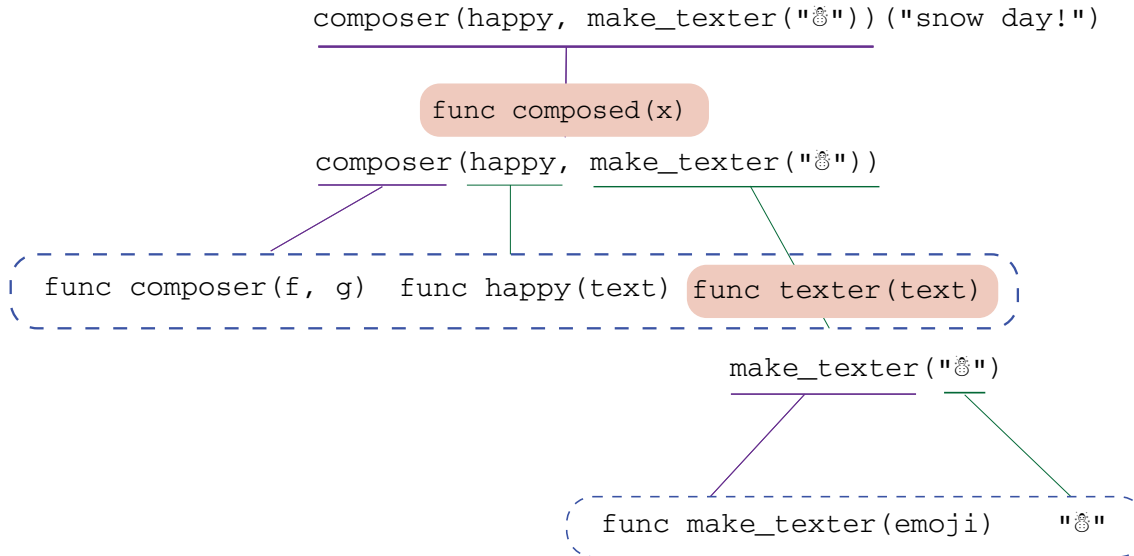
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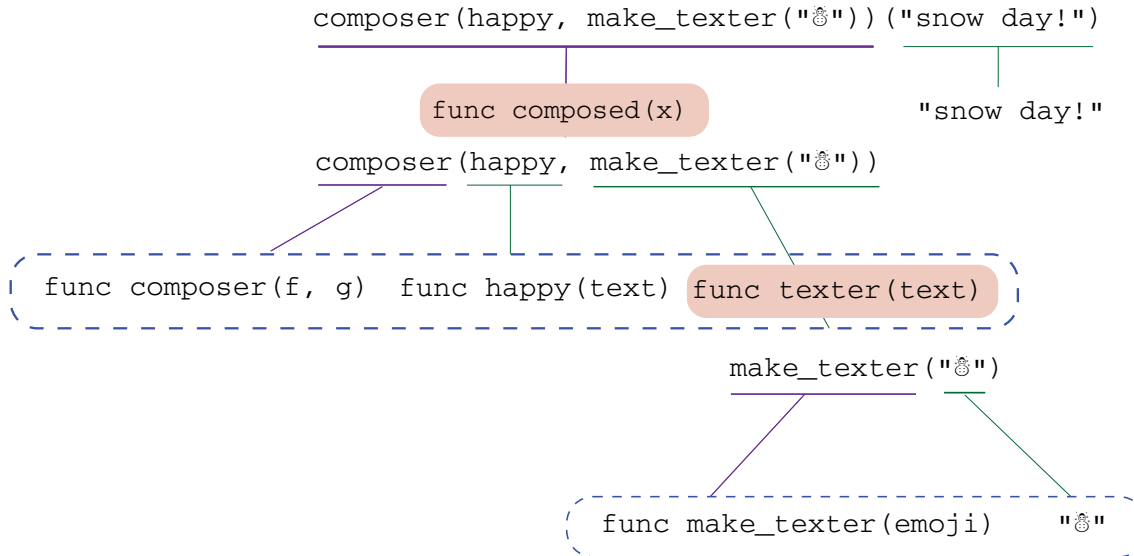
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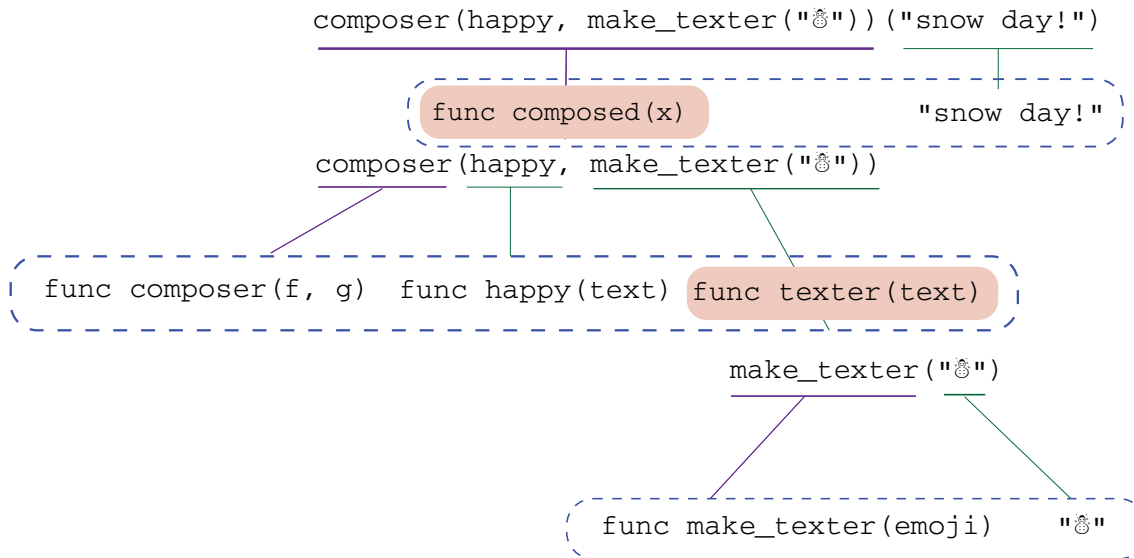
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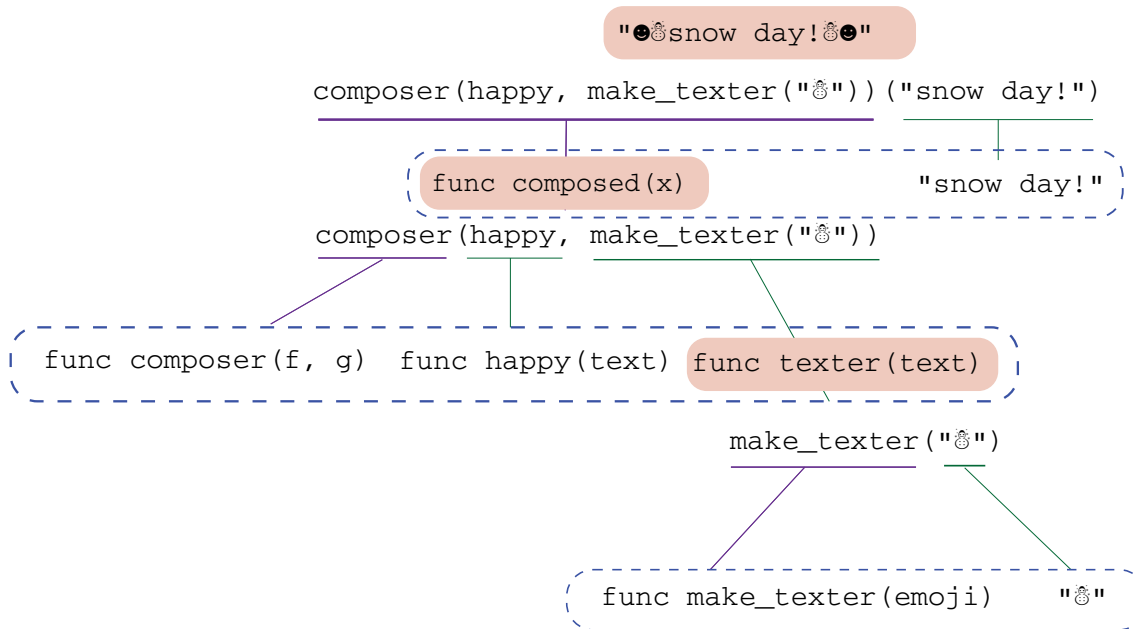
Composer 2 expression tree



Composer 2 expression tree



Composer 2 expression tree



Currying

add vs. make_adder

Compare...

```
from operator import add  
  
add(2, 3)
```

```
def make_adder(n):  
    return lambda x: n + x  
  
make_adder(2)(3)
```

What's the relationship between `add(2, 3)` and `make_adder(2)(3)`?

Function currying

Currying: Converting a function that takes multiple arguments into a single-argument higher-order function.

A function that currys any two-argument function:

```
def curry2(f):  
    def g(x):  
        def h(y):  
            return f(x, y)  
        return h  
    return g
```

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make_adder = curry2(add)  
make_adder(2)(3)
```

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        return h  
    return g
```

```
make_adder = curry2(add)  
make_adder(2)(3)
```

```
curry2 = lambda f: lambda x: lambda y: f(x, y)
```

Why "currying"?

It's not food! ✘ ✘

Named after American logician Haskell Curry, but actually published first by Russian Moses Schönfinkel, based on principles by German Gottlob Frege.

See also: [Stigler's law of eponymy](#)